UNCERTAIN VOLATILITY MODEL

Solving the Black Scholes Barenblatt Equation with the method of lines

GRM
Bernd Lewerenz

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Uncertain Volatility Model

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- In 1973 Black, Scholes and Merton published there Option Pricing Model which later earned them the nobel price.
- Following this paper it was now possible to price options without the need to know the real world drift of the underlying asset. They were constructing a risk less portfolio that growth at the risk free rate.
- The problem which remained was the estimation of the volatility which is unknown and definitely not a constant.
- Ever since then people have concentrated to model the remaining parameter “Volatility”.
- However in 1995 Avellanada proposed a radical different approach by accepting the fact the volatility is unknown In my talk I follow this path of Avellaneda and approach this problem from a sell side perspective using the uncertain volatiliy model.
- It calculates a conservative price for an option portfolio given the option writer is willing to take on some risk in the black scholes delta hedging strategy.

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Agenda

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A simple portfolio

Lets consider a portfolio consisting of 2 plain vanilla put options:

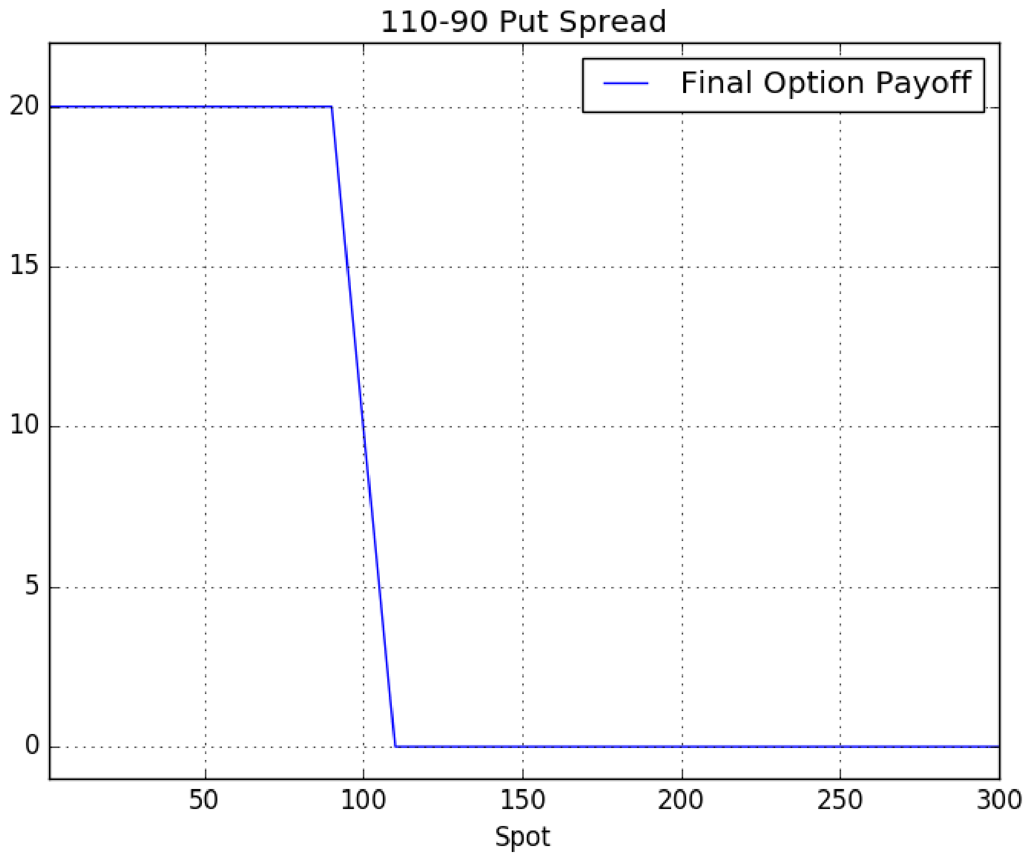
1. A long position in a put option stroke at 110 EUR.
2. A short position in a put option stroke at 90 EUR.

Both Options expire in 2 Years from now. The underlying of both options is a stock where we assume the following.

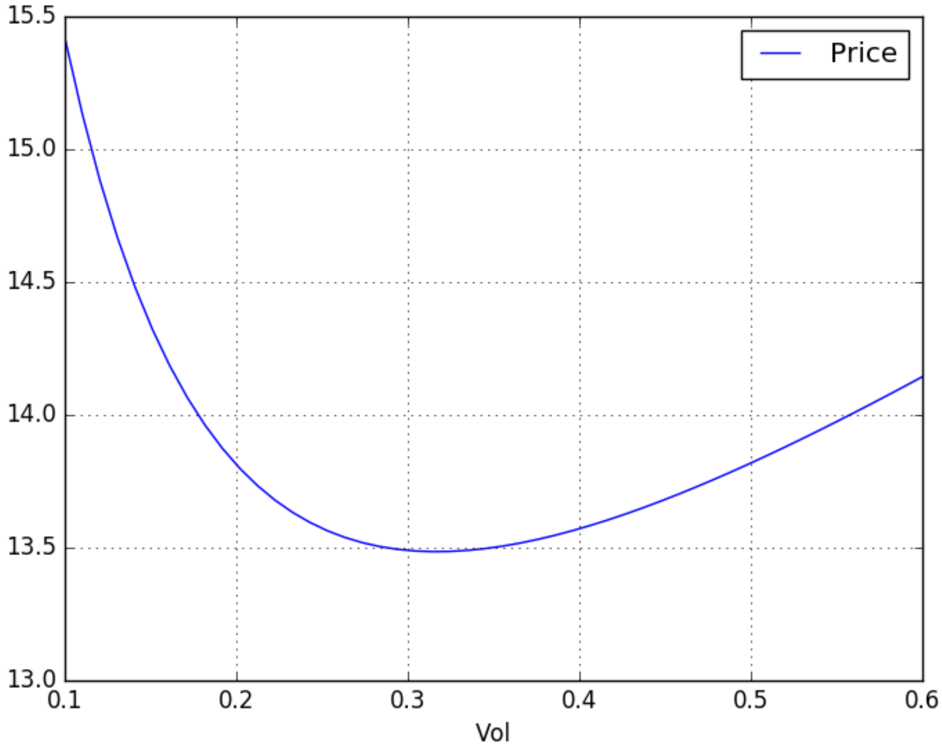
1) The riskless rate is at 0% and also the stock does not pay any dividends

2)We assume todays Spot Price of the Underlying beeing at 90 EUR

Payoff Profile of our Portfolio



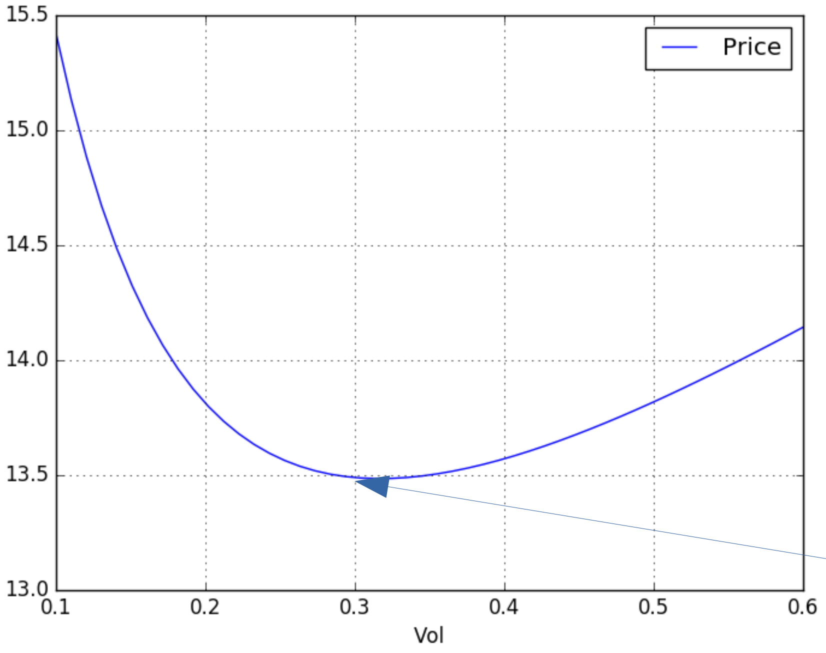
Sensitivity to Volatility (Black Model)



- We took the QL Black Model.
- Then pricing the put spread for a range of volatilities

Analysing the Put Spread

There is no implied
Black Volatitlity for
this portfolio



Here the vega is 0

The underlying assumptions of the BSB Equation

$$V^-(T,S)=F(S)$$

$$\Gamma=\frac{\partial^2 V^-}{\partial S^2}$$

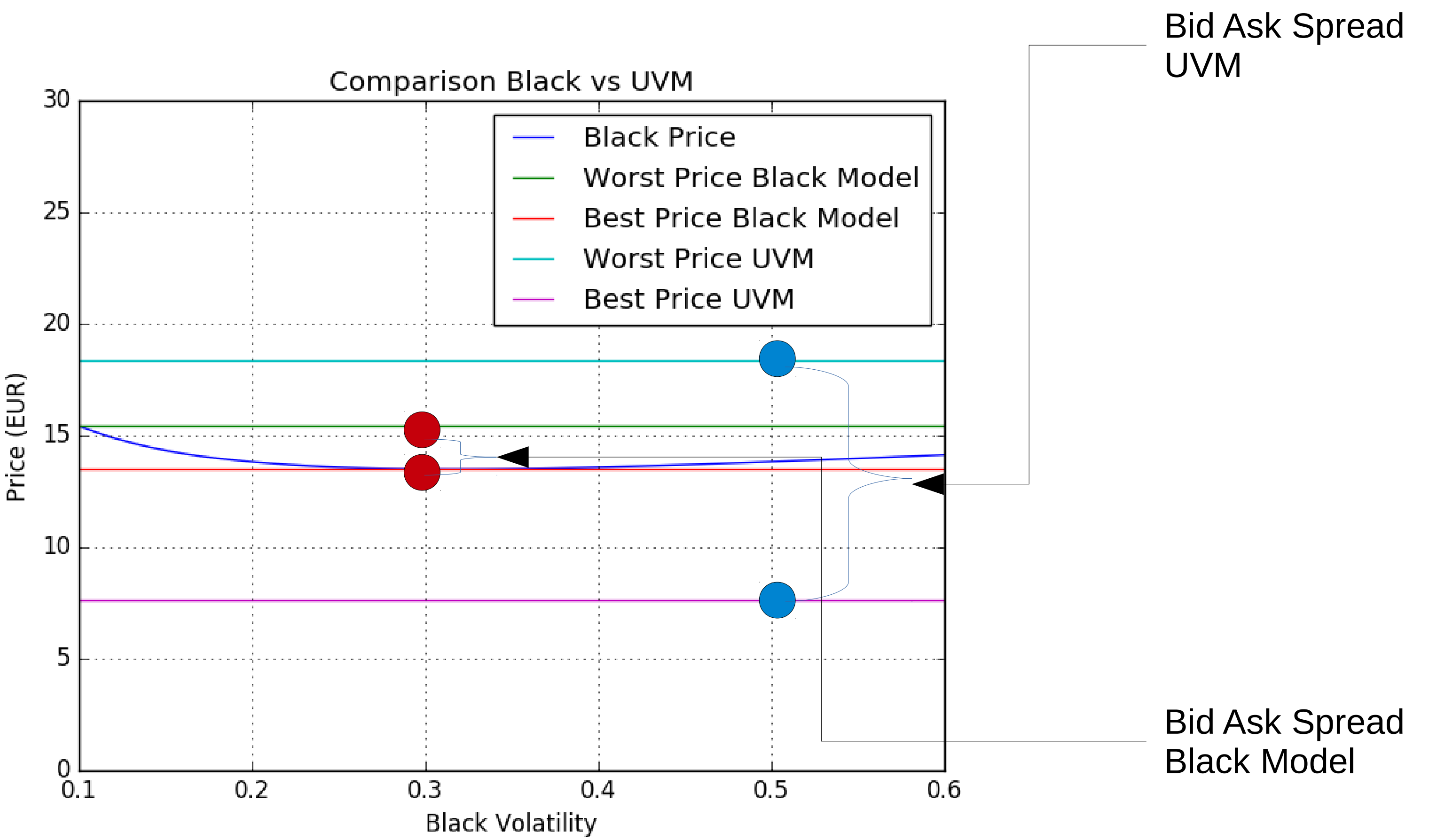
$$0=\frac{\partial V^-}{\partial t}+\frac{1}{2}*\sigma(\Gamma)^2*S^2*\frac{\partial^2 V^-}{\partial S^2}+r*S*\frac{\partial V^-}{\partial S}-rV^-$$

$$\sigma(\Gamma)=\begin{cases}\sigma^+ & if\ \Gamma\geq 0\\ \sigma^- & if\ \Gamma<0\end{cases}$$

$$\sigma^-\leq\sigma\leq\sigma^+$$

- In Wilmott(2006) is assumed to calculate a more conservative Price where a trader should sell the portfolio
- For calculating the best price (buy price) just interchange the roles of σ^+ and σ^- in the equation
- For a more rigorous mathematical derivation see Avellaneda (1995)

Key Portfolio Indicators



Numerical Solution of the BSB Equation with the method of lines

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There are 2 methods of discretization:

- **Space as shown in Hamdi (2007). This is the method we used here**
- **Time as shown in Meyer (2003). We wont show this method in this presentation**

$$0 = \underbrace{\frac{\partial V^-}{\partial t}}_{\text{Time}} + \frac{1}{2} * \sigma(\Gamma)^2 * S^2 * \underbrace{\frac{\partial^2 V^-}{\partial S^2}}_{\text{Space}} + r * S * \underbrace{\frac{\partial V^-}{\partial S}}_{\text{Space}} - r V^-$$

The aim is to transform this PDE in a system of coupled ODEs which we then solve using the software in Ahnert (2011)

$$\frac{dV^-}{dt} = f(t, V^-)$$

Operator for ODEint

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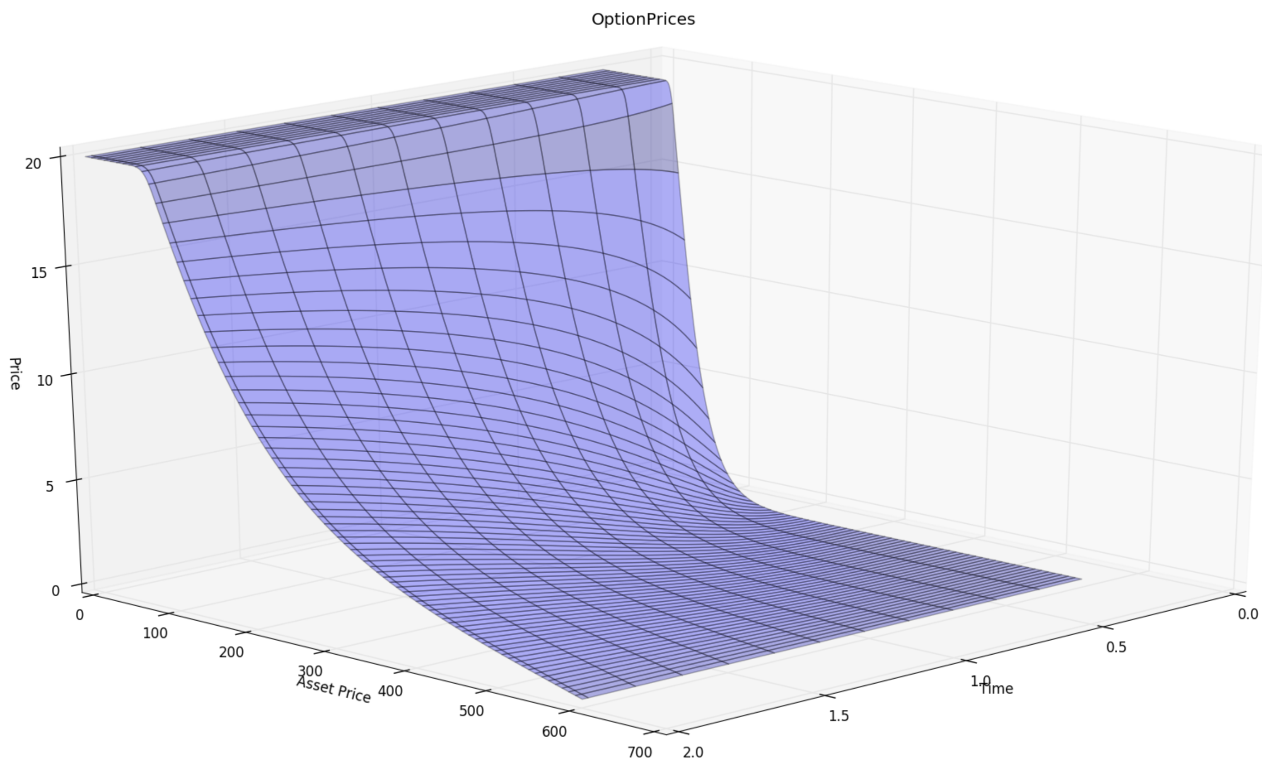
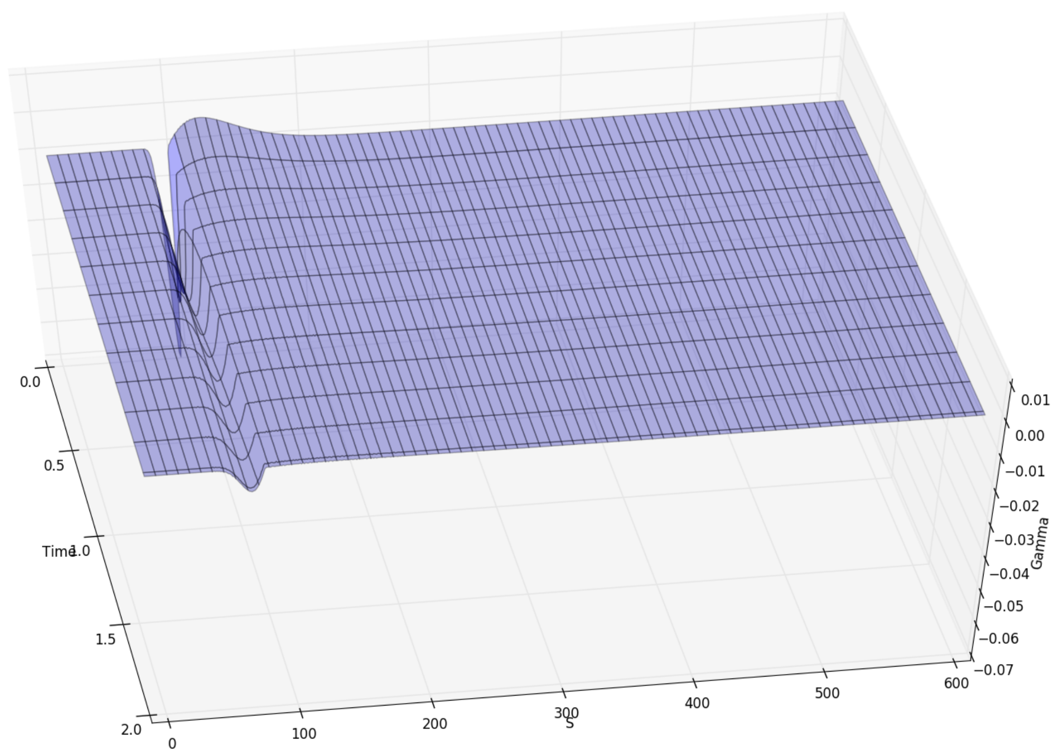
```
File Edit Options Buffers Tools C++ Help

struct bsb_operator
{
  bsb_operator(double r_, double q_, double h_,double s_min_,double sigma_low_,double sigma_high_,UVM price_)
    :h(h_),q(q_),r(r_),s_min(s_min_),sigma_low(sigma_low_),sigma_high(sigma_high_),price(price_){};

  // Keep the paramters of asset price and pde here
  double h,r,q,s_min,sigma_low,sigma_high;
  double s; // holds Assetprice
  UVM price; // Enumerator for Type of Price (WorstPrice or BestPrice)
  void operator()( const vector_type &x , vector_type &dxdt , double /* t */ )
  {
    double sigma;
    int N_s = x.size()-1;


    for (int i =1; i<=N_s-1;i++ )
      {
        double delta = (x[ i+1 ] - x[ i-1 ]) / (2*h); // FD Approximation for Delta
        double gamma = ( x[i+1] -2 * x[i] + x[i-1])/pow(h,2); // Approximation for Gamma
        if (gamma <0.0) // This is the difference to Black Scholes Implements sigma(Gamma)
          {
            if (price == UVM::WorstPrice){
              sigma = sigma_low;
            } else
              {
                sigma = sigma_high;
              }
          } else
          {
            if (price == UVM::WorstPrice){
              sigma = sigma_high;
            } else {
              sigma = sigma_low;
            }
          }
        s = s_min + i *h;
        // Calculate the Option Theta using the pde
        dxdt[ i ] = (r-q) * s * delta +
          0.5 * pow(sigma,2) * pow(s,2) * gamma
          - r * x[i];

      }
    dxdt[N_s] = -r * x[N_s-1];
    //dxdt[0] = (q-r) * s_min - r * x[0];
    dxdt[0] = - r * x[0];

  }
};

-:---  bsb_put_spread.cpp    15% L51    (C++/l Abbrev)
```

- Solving the BSB Equation with the method of lines

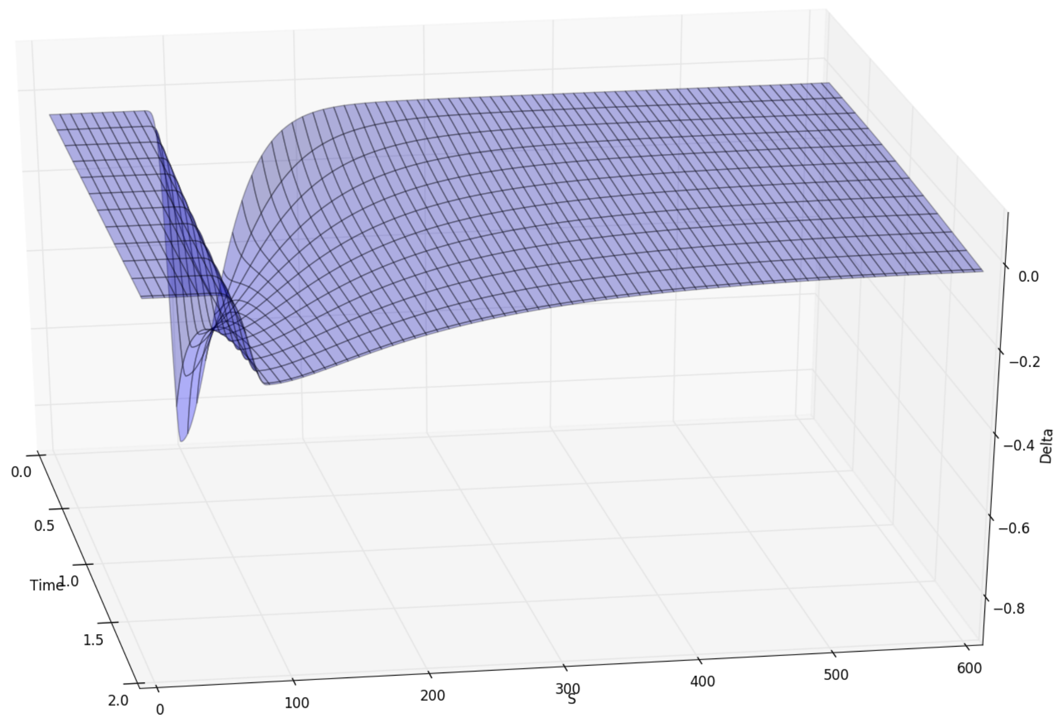
UVM Worst Price Surface



UVM Worst Price Gamma Surface



UVM Worst Price Delta Surface



Perspective

- The UVM PDE in non linear
- The present QuantLib offers pricing on an instrument level only and can not handle portfolio effects as presented by the UVM (or CVA models)
- What could be further steps to raise the QL capacity to portfolio level?

Numerical Details and Hardware

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- Grid has 600 Steps in Asset Price Direction.
- The grid spans from 1 EUR to 600 EUR in Asset Direction.
- The solution inside the odeint integration at time steps of 0.02 Years.
- The Bulirsch Stoer Stepper from ODEINT to integrate the ode system.
- It took about 1.5 seconds to solve for the price. With 300 Asset Steps it took 0.25 seconds
- The calculations were run on a pc running ubuntu.
- The processor has been an intel i7-7700K with 16 GB of RAM

The Odeint Library within boost

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- We use our nice friend library boost. It ships with a nice numerical package named odeint.
- The authors of Odeint were Karsten Ahnert and Mario Mulansky.
- There is a lot of documentation to be found at http://www.odeint.com

References

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