ASPECTS OF PRICING IRREGULAR SWAPTIONS WITH QUANTLIB

Calibration and Pricing with the LGM Model

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European und Bermudan Swaptions: Appearance

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Swaptions (options on interest rate swaps) serve as building blocks in the context of:

- callable fixed rate bonds
- callable zero bonds
- callable loans (i.e. German law: §489 BGB)
- hedges of callable loans and bonds
- EPE and ENE profiles of swaps for the computation of XVA

...

Irregular swaptions appear in a natural way for example by constructing:

- callable zero bonds
- EPE and ENE profiles of irregular swaps

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Regular European Swaptions

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„Regular“ could be interpreted that the swaption has the features of the broker. Conventions depend on currencies. Typical conventions for EUR swaption volatilities are:

- fix basis: 30/360, fix frequency: 12m
- float basis: Actual/360, float frequency: 3m (maturity=1 year), 6m (maturity>1 year)
- calendar: TARGET
- adjustment: modified following
- settlement style: cash

Broker quotes of implied volatilities follow typically a swap market model.

Swap market model solution for an European swaption is the well known Black76 pricing formula.

QuantLib engine: BlackSwaptionEngine

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Irregular European Swaptions

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Irregularities are then in principle all features of the underlying swap or the swaption, that do not match the conventions of the broker:

- step-up coupon on fixed side
- (step-up) spread on the float side
- nominal structure
- notification period <> spot period
- exercise fees

...

Irregular European Swaptions in the LGM Model

Need of a model to calibrate against swaption prices. Choice: Linear Gauss Markov model (LGM model):

- model is technically comfortable
- multicurve extension is easy to implement
- parameterization is connected to the Hull-White model (HW model)
- can be found in front office systems

Alternatives:

- HW model in the HJM framework

...

Swap and Swaption in the LGM Model: Basics

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LGM model, process of „state variable“:

$$dX = \alpha(t)dW \qquad \text{with} \qquad X(0) = 0$$

LGM model, numeraire:

$$N(t,x) = \frac{1}{D(t)} \exp(H(t)x + \tfrac{1}{2} H^2(t)\zeta(t))$$

with $\zeta(t) = \int_0^t \alpha^2(s)ds$

Swap and Swaption in the LGM Model: Basics

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Connection to the HW model:

$$H(t) = (1 - e^{-\kappa * t}) / \kappa$$

$$\zeta(t) = \frac{\sigma^2}{2\kappa}(e^{2*\kappa*t} - 1)$$

The HW short rate volatility is chosen to be constant for simplicity.

Swap and Swaption in the LGM Model: Swap

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Swap is characterized by its fixed leg.

Assume a given basis- or coupon spread on the floating leg $S_{i,\,float}$.

The transformed spread is then given by (the nominal is assumed to be constant):

$$S = \sum_{i=1}^{n} \tau_{i,\,float} S_{i,\,float} D(t_i) \Big/ A_{fix}$$

with

$$A_{fix} = \sum_{i=1}^{m} \tau_{i,\,fix} D(t_i)$$

Swap and Swaption in the LGM Model: Swaption

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Times: $t_{ex}, t_0, \ldots, t_i, \ldots, t_n$, fixed rate: R , spread: S

Parameters: $H_i = H(t_i)$ and $\zeta_{ex} = \zeta(t_{ex})$

Discount curve: $D_i = D(t_i)$

Swaption value (payer):

$$NPV_{Swaption} = D_0 N\left(\frac{-y^*}{\sqrt{\zeta_{ex}}}\right) - D_n N\left(\frac{-y^* - (H_n - H_0)\zeta_{ex}}{\sqrt{\zeta_{ex}}}\right) - \sum_{i=1}^{n} \tau_i (R-S) D_i N\left(\frac{-y^* - (H_i - H_0)\zeta_{ex}}{\sqrt{\zeta_{ex}}}\right)$$

State variable y^* has to fulfill (defines „break even“ point of state variable):

$$D_0 = D_n \exp\left(-y^*(H_n - H_0) - 1/2(H_n - H_0)^2 \zeta_{ex}\right) - \sum_{i=1}^{n} \tau_i (R-S) D_i \exp\left(-y^*(H_i - H_0) - 1/2(H_i - H_0)^2 \zeta_{ex}\right)$$

We focus on an option on a bullet swap with nominal=1 here.

Swap and Swaption in the LGM Model: Calibration

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The model is going to be fixed via 2 loops:

- Inner: Find state variable y^* by solving the break even equation.
- Outer: Fixing of parameters $H_i = H(t_i)$ and $\zeta_{ex} = \zeta(t_{ex})$ by fit to swaption prices of **proper** calibration instruments.

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Options on Irregular Swaps: Calibration

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In the case of a „regular“ swaption one can work with the implied volatilities and the corresponding Black76 NPVs.

What would be the right volatility (and the NPV) in the case of an „irregular“ swaption?

Options on Irregular Swaps: Calibration

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Payoff Matching

The original irregular underlying swap is replicated by a regular swap.

The parameters nominal, tenor and strike are used to fit the NPV, delta and gamma of the original swap.

The resulting swap is used to find the Black76 volatility. The NPV can be found pricing the swaption of the replicating regular swap via Black76.

Options on Irregular Swaps: Calibration

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Basket

Cash flows of the underlying swap (interest R_i and nominal payments $N_{i-1} - N_i$) are replicated by a set of n regular swaps (the „basket") with weights C_i .

The begin date is identical for all swaps: $T_{Begin,i} = t_0$

The end date is for the i-th swap: $T_{End,i} = t_0 + \frac{i}{n}(t_n - t_0)$

The fixed rate of the i-th swap consists of the individual fair rate r_i plus a global parameter lambda λ: The i-th swap has fixed rate $r_i + \lambda$.

Lambda is fixed by the condition, that the initial nominal N_0 of the original swap is equal to the weighted initial nominal of the basket components.

The result is a vector of weights C_i and an „add on" lambda λ . For each of the corresponding swaptions a Black76 volatility can be found straightforward.

The NPV proxy could be the weighted sum of the NPVs of the basket swaptions (talk by A. Miemiec, 2013). QuantLib engine: HaganIrregularSwaptionEngine

Options on Irregular Swaps: Calibration

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Basket + LGM with HW Parameterization

The result from basket calibration is a set of n swaps with fixed rates $r_i + \lambda$ and weights C_i .

Since all basket swaps are regular, we can price the swaptions via Black76.

The LGM model in the HW parameterization has two parameters, choose the mean reversion for example at 1,5%.

Ansatz

The remaining parameter (HW short rate volatility) could be fixed via a least squares approach:

$$\min\left\{\sum_{i=1}^{n} C_i^{\,2}\left(NPV_i^{\,B76} - NPV_i^{\,LGM}\right)^2\right\}$$

The result is the short rate volatility in the HW model.

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Irregular Bermudan Swaptions (HW)

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HW model (Hull-White ext. Vasicek):

$$dr = \left[\theta(t) - \kappa(t) r\right] dt + \sigma(t) dW_t$$

Evaluation of Bermudans with a tree: piecewise constant HW short rate volatilities needed

QuantLib engine/model: TreeIrregularSwaptionEngine/GeneralizedHullWhite

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Example 1: Regular European Swaption

„Regular“ European swaption, 2% vs. 6M

Start/end: 8.8.2024/8.8.2034

Exercise: 6.8.2024

Evaluation date: 4.1.2016

Nominal=1,5 Mio. EUR

Single curve, SABR volatility

Result	Engine/Model	Difference to BlackSwaptionEngine in bp related to Nominal
119.075,09	BlackSwaptionEngine, NPV	
119.076,65	HaganIrregularSwaptionEngine (Basket), NPV	0,01
119.079,15	Basket plus LGM with HW Parameterization, NPV	0,03
0,8246209	Basket plus LGM with HW Parameterization, HW Volatility in %	
119.089,15	TreeIrregularSwaptionEngine/GeneralizedHullWhite/TimeSteps=4000, NPV	0,09
119.079,15	JamshidianSwaptionEngine/HullWhite, NPV	0,03

Example 2: Irregular European Swaption

„Irregular" European swaption, 2% vs. 6M

Start/end: 8.8.2024/8.8.2034

Exercise: 6.8.2024

Evaluation date: 4.1.2016

Nominal=1,5 - 4,16 Mio. EUR, nominal is increasing 12% p.a.

Single curve, SABR volatility

Result	Engine/Model	Difference to Basket in bp related to lowest Nominal	
n.a.	BlackSwaptionEngine, NPV		
206.631,89	HaganIrregularSwaptionEngine (Basket), NPV		
203.304,83	Basket plus LGM with HW Parameterization, NPV	-	22,18
0,8240310	Basket plus LGM with HW Parameterization, HW Volatility in %		
203.312,05	TreeIrregularSwaptionEngine/GeneralizedHullWhite/TimeSteps=4000, NPV	-	22,13
n.a.	JamshidianSwaptionEngine/HullWhite, NPV		

Implementation

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Code is in „prove of concept“ state.

Extensions are accommodated within class HaganIrregularSwaptionEngine:

- translating HW parameters to LGM parameters
- LGM implementation (together with A. Miemiec)
- modified Excel (*.qlo) and new Python (*.i) interface

Next steps could be:

- extension to piecewise constant HW short rate volatilities (for Bermudans)
- separate LGM model (model is already implemented in ORE)
- extension to other calibration schemes

Summary

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- Swaptions are basic components in many settings.
- Quotation, calibration and evaluation can be performed separately and eventually in different models.
- **Calibration** is not unique for nonstandard instruments: There is an additional degree of freedom in pricing besides (for example) the choice of the **mean reversion** for Bermudans.
- QuantLib provides many possibilities for pricing of complex swaptions. We presented one possible scheme to compute the NPV and the HW model parameters of an irregular European swaption with a QuantLib prototype.
- Results can be extended to the multicurve- and the Bermudan case easily.

Literature

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- Björk: Arbitrage Theory in Continuous Time, third edition (2009)
- Hagan: Evaluating and Hedging Exotic Swap Instruments via LGM
- Hagan: Methodology for Callable Swaps and Bermudan „Exercise into“ Swaptions
- Miemiec: Pricing of Accreting Swaptions using QuantLib, talk given at QuantLib User Meeting 2013